11. $\lim_{n \to \infty} \sum_{k=1}^{n} \left[2 + \frac{3}{n}k\right]^2 \left(\frac{3}{n}\right) =$

(A) 13 (B) $\frac{125}{3}$ (C) 39 (D) 125

43. If $f(x) = e^x \ln x$, then $f'(e) =$

(A) $e^{e+1} + e^e$

(B) $e^{e-1} + e^e$

(C) $e^e + e$

(D) $e^e + \frac{1}{e}$

5. What is the radius of convergence for the power series $\sum_{n=0}^{\infty}(3x-5)^n$?

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) $\frac{5}{3}$

9. Let a_n, b_n, and c_n be sequences of positive numbers such that for all positive integers n, $a_n \le b_n \le c_n$. If $\sum_{n=1}^{\infty} b_n$ converges, then which of the following statements must be true?

I. $\sum_{n=1}^{\infty} a_n$ converges

II. $\sum_{n=1}^{\infty} c_n$ converges

III. $\sum_{n=1}^{\infty} (a_n + b_n)$ converges

(A) II only

(B) III only

(C) I and III only

(D) I, II, and III

22. The Taylor series for $\dfrac{e^{2x}-1}{x}$ centered at $x=0$ is

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{2(x)^{n-1}}{n!}$

(B) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(-x)^{n-1}}{n!}$

(C) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(x)^{n-1}}{n!}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(x)^{n-1}}{(n-1)!}$

41. The graph of the third-degree Maclaurin polynomial for $\sin(x)$ intersects the gaph of $y = x^2 - 1$ at approximately

(A) 0.879 (B) 1.066 (C) 1.262 (D) 1.394

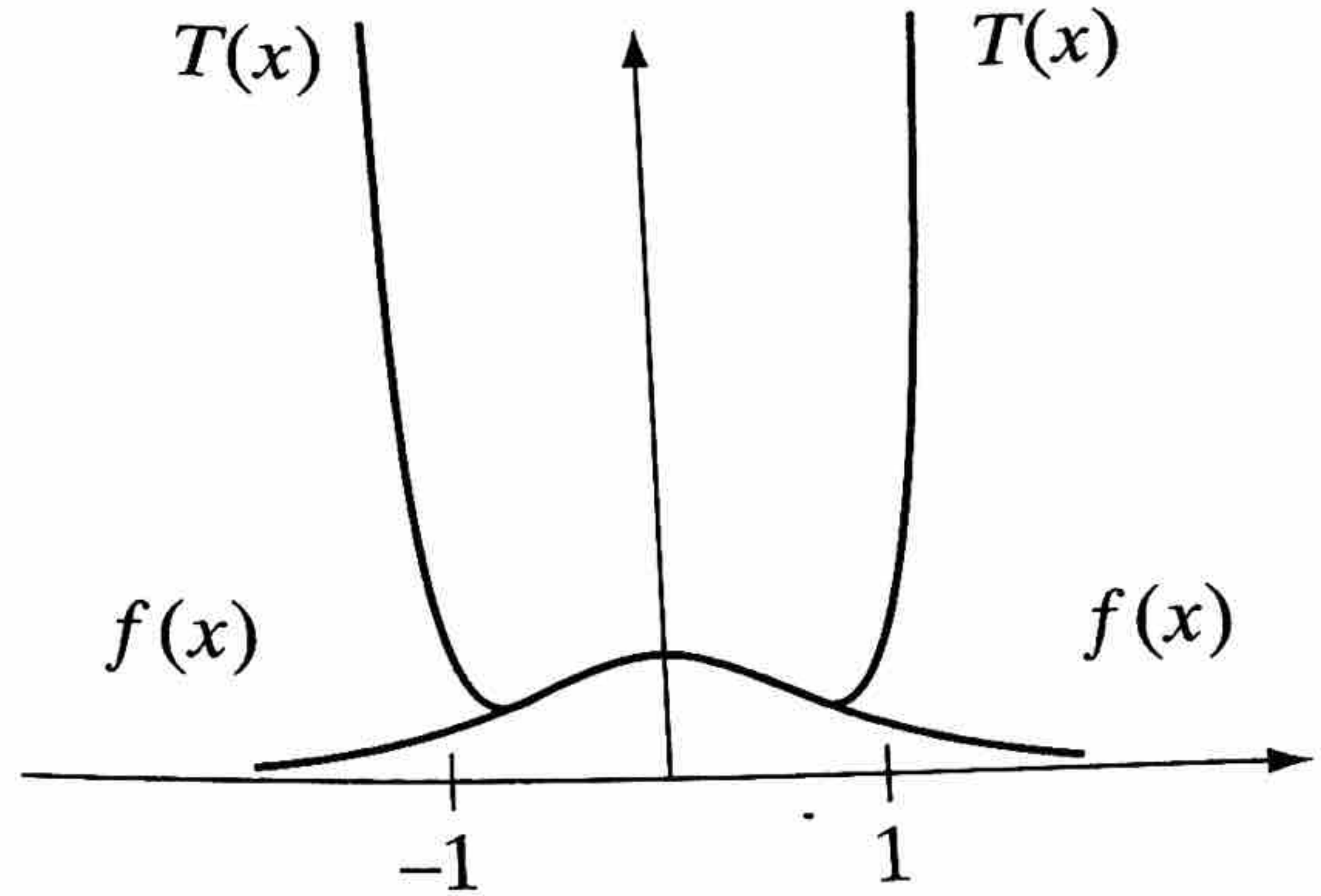
7. What is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{6^n}$?

(A) $\frac{\sqrt{6}}{2}$

(B) $\sqrt{6}$

(C) $2\sqrt{6}$

(D) 6



10. The figure above shows the graph of $y = f(x)$ and $y = T(x)$ where $T(x)$ is a Taylor poly for $f(x)$ centered at zero. Which of the following statements must be true?

I. $T(0.5)$ is a good approximation for $f(0.5)$.

II. $T(1.5)$ is a good approximation for $f(1.5)$.

III. $T(0) = f(0)$

(A) I only

(B) III only

(C) I and II only

(D) I and III only

12. Which of the following improper integrals converge?

I. $\int_0^\infty e^{-x}\,dx$

II. $\int_0^1 \frac{1}{x^2}\,dx$

III. $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

(A) I only

(B) III only

(C) II and III only

(D) I and III only

17. The first three nonzero terms in the Maclaurin series of xe^{-x} are

(A) $x - x^2 - \frac{x^3}{2!}$

(B) $x - x^2 + \frac{x^3}{2!}$

(C) $x + x^2 + \frac{x^3}{2!}$

(D) $1 - x + \frac{x^2}{2!}$

19. $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{2n}$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

21. The Taylor series for $\frac{\sin(x^2)}{x^2}$ centered at $x = 0$ is

(A) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$

(B) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k+1)!}$

(C) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k)!}$

(D) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{4k}}{(2k+1)!}$

24. Which of the following series are convergent?

I. $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\cdots+\frac{1}{n\sqrt{n}}+\cdots$

II. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}+\cdots$

III. $1+\frac{1}{\ln 2}+\frac{1}{\ln 3}+\cdots+\frac{1}{\ln(n+1)}+\cdots$

(A) I only

(B) II only

(C) I and II only

(D) I, II, and III

25. For all x if $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$, then $f'(x) =$

(A) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n}}{(2n+1)!}$

(B) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n}}{(2n)!}$

(C) $\sum_{n=0}^{\infty} \frac{(-1)^{n} x^{2n}}{(2n)!}$

(D) $\sum_{n=0}^{\infty} \frac{(-1)^{n} x^{2n}}{(2n+1)!}$

29. For the series $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$ which of the following statements is true?

(A) The series diverges.

(B) The series converges absolutely.

(C) The series converges conditionally

(D) The series converges but not absolutely nor conditionally.

14. $\sum_{k=0}^{\infty}\left(-\frac{\pi}{3}\right)^{k}$ is

(A) $\frac{1}{1-\frac{\pi}{3}}$ (B) $\frac{1}{1+\frac{\pi}{3}}$ (C) $\frac{\frac{\pi}{3}}{1+\frac{\pi}{3}}$ (D) divergent

27. $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ will

(A) converge by the n^{th} term test.

(B) converge by the Alternating Series Test.

(C) diverge by the Ratio Test.

(D) diverge by the Limit Comparison Test.

34. Let f be the function given by $f(x) = \int_0^x \cos \sqrt{t}\, dt$, $x \geq 0$.

Which of the following is the Taylor series of f about $x = 0$

(A) $1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720} + \cdots$

(B) $x - \frac{x^2}{2} + \frac{x^4}{6} - \frac{x^6}{24} + \cdots$

(C) $x - \frac{x^2}{3} + \frac{x^4}{15} - \frac{x^5}{105} + \cdots$

(D) $x - \frac{x^2}{4} + \frac{x^3}{72} - \frac{x^4}{2880} + \cdots$

38. Let f be a function that is everywhere differentiable. The table below provides information about $f(x)$ and its first, second, and third derivatives for selected values of x.

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	4	2	1	0.50
1	5.15	2.50	1.25	0.75
2	7.20	3.50	1.75	0.85
3	10.50	5.25	2.10	1.00

Which of the following best approximates $f(2.2)$?

(A) $4+2(0.2)+\frac{1(0.2)^2}{2}+\frac{0.5}{6}(0.2)^3$

(B) $4+2(2.2)+\frac{1}{2}(2.2)^2+\frac{0.50}{6}(2.2)^3$

(C) $7.20+3.50(2.2)+\frac{1.75}{2}(2.2)^2+\frac{0.85}{6}(2.2)^3$

(D) $7.20+3.50(0.2)+\frac{1.75}{2}(0.2)^2+\frac{0.85}{6}(0.2)^3$

39. Let E be the error when the Taylor polynomial $T(x) = x - \frac{x^3}{3!}$, centered about $x = 0$, is used to approximate $f(x) = \sin x$ at $x = 0.5$. Which of the following is true?

(A) $0.0001 < |E| < 0.0003$

(B) $0.0003 < |E| < 0.0005$

(C) $0.0005 < |E| < 0.0007$

(D) $0.0007 < |E|$

45. What is the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n (n+1)}$?

(A) $-1 \leq x < 5$

(B) $-1 < x \leq 5$

(C) $-1 \leq x \leq 5$

(D) $-1 < x < 5$

5. What are all values of x for which $\sum_{n=1}^{\infty} \frac{2^n x^n}{n}$ converges?

(A) $-\frac{1}{2} \leq x \leq \frac{1}{2}$

(B) $-\frac{1}{2} < x < \frac{1}{2}$

(C) $-\frac{1}{2} < x \leq \frac{1}{2}$

(D) $-\frac{1}{2} \leq x < \frac{1}{2}$

23. If $s_n = \left(\frac{(8-n)^{200}}{8^{n+2}}\right)\left(\frac{8^n}{(3-n^2)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge as $n \to \infty$?

(A) $-\frac{1}{8}$

(B) $-\frac{1}{64}$

(C) $\frac{1}{64}$

(D) $\frac{1}{8}$

39. Let $f(x)$ be a function that is differentiable for all x. The derivative of this function is given by the power series

$$f'(x) = 3x - \frac{9x^3}{2} + \frac{81x^5}{40} - \frac{3x^7}{60} + \cdots$$

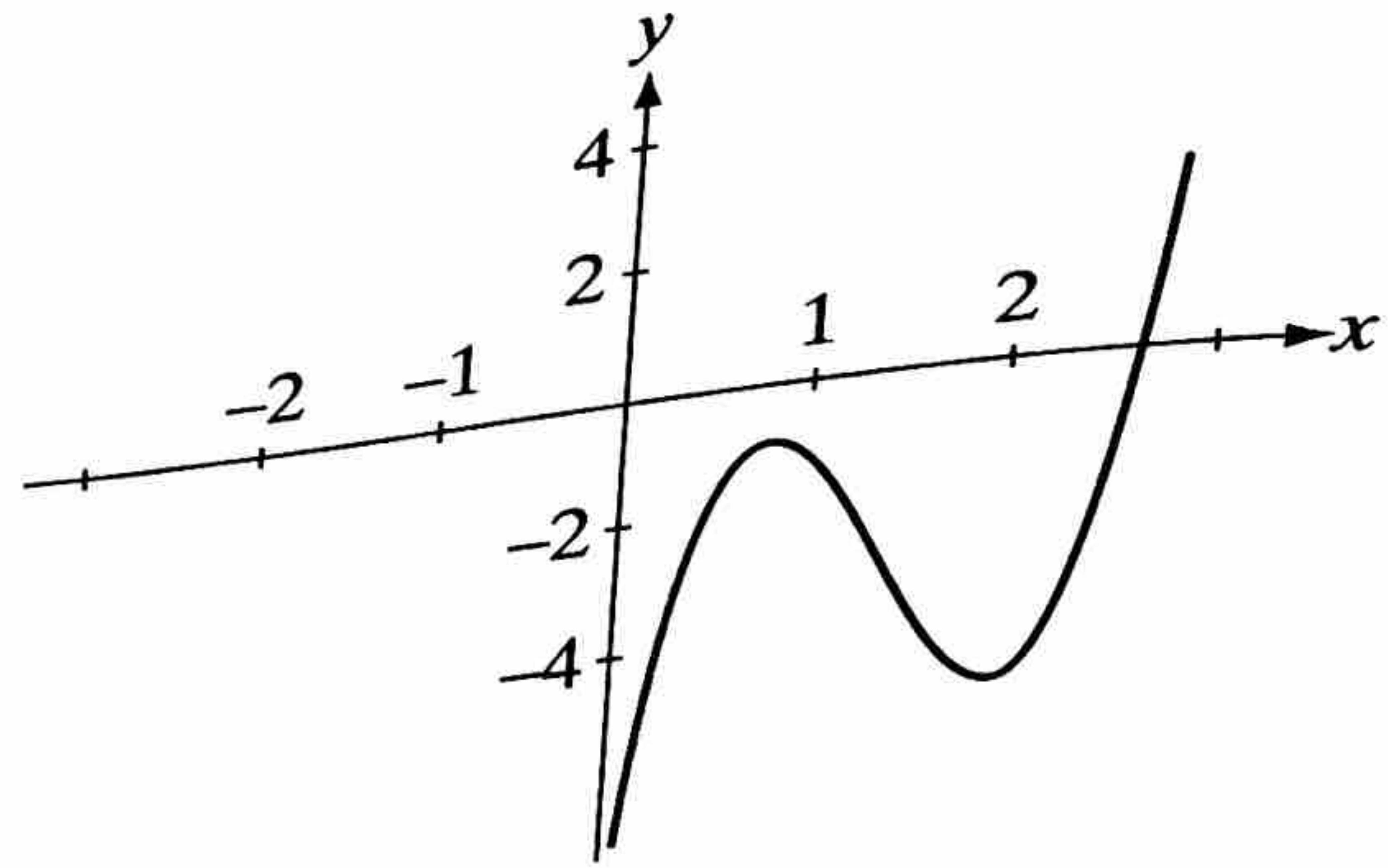
If $f(0) = 2$, then $f(x) =$

(A) $2 + 3x - \frac{9x^3}{2} + \frac{81x^5}{40} - \frac{3x^7}{60} + \cdots$

(B) $\frac{3x^2}{2} - \frac{9x^4}{8} + \frac{27x^6}{80} - \frac{3x^8}{480} + \cdots$

(C) $2 - \frac{3x^2}{2} + \frac{9x^4}{8} - \frac{27x^6}{80} + \frac{3x^8}{480} + \cdots$

(D) $2 + \frac{3x^2}{2} - \frac{9x^4}{8} + \frac{27x^6}{80} - \frac{3x^8}{480} + \cdots$



44. The graph above shows a function f with a relative minimum at $x = 2$. The approximation of $f(x)$ near $x = 2$ using a second-degree Taylor polynomial centered about $x = 2$ is given by $a + b(x-2) + c(x-2)^2$.

Which of the following is true about a, b, and c ?

(A) $a < 0, b = 0, c > 0$

(B) $a > 0, b = 0, c < 0$

(C) $a < 0, b > 0, c > 0$

(D) $a > 0, b = 0, c > 0$

36. If $\sum_{n=1}^{\infty} |a_n|$ converges, then which of the following is true?

I. $\sum_{n=1}^{\infty} a_n$ converges.

II. $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

III. $\sum_{n=1}^{\infty} -a_n$ converges.

(A) I only

(B) II only

(C) III only

(D) I, II, and III

38. Let f be a function whose seventh derivative is $f^{(7)}(x) = 10{,}000 \cos x$ where $x = 1$ is in the interval of convergence of the power series for this function. Using the Lagrange error bound the Taylor polynomial of degree six centered at $x = 0$ will approximate $f(1)$ with an error of not more than

(A) 1.98×10^{-4}

(B) 3.21×10^{-2}

(C) 1.072

(D) 1.984

Let $f(x)$ be a function whose Taylor series converges for all x. If $\left|f^{(n)}(x)\right| < 1$ where $f^{(n)}(x)$ is the n^{th} derivative of $f(x)$, what is the minimum number of terms of the Taylor series centered at $x = 1$ necessary to approximate $f(1.2)$ with a Lagrange error less than 0.00001 ?

(A) Three

(B) Four

(C) Five

(D) Six

40. Let $T(x) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \frac{(x-3)^k}{k!}$ be the Taylor series for a function f.

What is the value of $f^{(10)}(3)$, the tenth derivative of f at $x = 3$?

(A) 5.382×10^{-10}

(B) 2.691×10^{-10}

(C) 9.766×10^{-4}

(D) 4.883×10^{-4}

43. What is the approximate value of $\cos\left(\frac{1}{2}\right)$ obtained by using a fourth-degree Taylor polynomial for $\cos x$ about $x = 0$?

(A) $\frac{1}{2} - \frac{1}{24} + \frac{1}{640}$

(B) $1 - \frac{1}{4} + \frac{1}{16}$

(C) $1 - \frac{1}{8} + \frac{1}{64}$

(D) $1 - \frac{1}{8} + \frac{1}{384}$

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