3. For the line tangent to the graph of C to have a slope of 4 means $\frac{dy}{dx} = 4$.

Thus $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-3}{4t+1} = 4$

Solving $\frac{2t-3}{4t+1} = 4$, $4(4t+1) = 2t-3$ or $16t+4 = 2t-3$, and $t = -\frac{1}{2}$.

The correct choice is (A).

15. Integrate the velocity function $v(t)$ to find the position function $y(t)$:

$$y(t) = \int v(t)\, dt = \int (8 - 2t)\, dt = 8t - t^2 + C$$

The velocity is positive when $t < 4$ since during this time the particle is moving upwards.

At $t = 4$, $v(4) = 0$ and the particle stops moving up when it reaches the origin and begins moving down, therefore $y(4) = 0$.

$$y(4) = 8(4) - 4^2 + C = 0 \Rightarrow C = -16$$

Therefore, the position function is $y(t) = -t^2 + 8t - 16$.

The correct choice is (A).

17. $e^x = 1 + x + \frac{x^2}{2!} + \cdots$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \cdots$$

$$xe^{-x} = x\left(1 - x + \frac{x^2}{2!} - \cdots\right) = x - x^2 + \frac{x^3}{2!} - \cdots$$

The correct choice is (B).

30. The acceleration becomes negative when the velocity stops increasing and starts decreasing (even though the velocity may still be positive). This corresponds to a point of inflection on the graph, where the concavity changes from up (on the left) to down (on the right). Point C appears to be the closest point where there is a point of inflection.

The correct choice is (C).

31. Differentiate the position equations to find the velocity vector:

$v(t) = \langle -9 \sin t, 4 \cos t \rangle$.

Then differentiate the velocity vector to find the acceleration vector:

$a(t) = \langle -9 \cos t, -4 \sin t \rangle$

Substitute the value $t = 3$, $a(3) = \langle -9 \cos(3), -4 \sin(3) \rangle = \langle 8.190, -0.564 \rangle$.

The correct choice is (C).

38. The length of the path $= \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\frac{dx}{dt} = \cos t$ and $\frac{dy}{dt} = -2 \sin(2t)$

Length $= \int_0^{2\pi} \sqrt{\cos^2 t + 4 \sin^2 (2t)}\, dt$

Use a calculator to find the length to be 9.294.

The correct choice is (B).

4. The velocity vector is found by finding $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

$\frac{dx}{dt} = (e^t)(\cos t) + (\sin t)(e^t)$ and $\frac{dy}{dt} = (e^t)(-\sin t) + (\cos t)(e^t)$

At $t = \pi$, $\frac{dx}{dt} = (e^{\pi})(\cos \pi) + (\sin \pi)(e^{\pi}) = -e^{\pi} + 0 = -e^{\pi}$

At $t = \pi$, $\frac{dy}{dt} = (e^{\pi})(0) + (-1)(e^{\pi}) = -e^{\pi}$

Therefore, the velocity vector $v(t) = \langle -e^{\pi}, -e^{\pi} \rangle$.

The correct choice is (D).

9. The slope of the tangent line, $\frac{dy}{dx}$, equals $\frac{dy}{d\theta} \div \frac{dx}{d\theta}$, where $y = r\sin(\theta)$ and $x = r\cos(\theta)$.

$$\frac{dy}{dx} = \frac{\cos(2\theta)\cos(\theta) - 2\sin(\theta)\sin(2\theta)}{-\cos(2\theta)\sin(\theta) - 2\cos(\theta)\sin(2\theta)}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = \frac{\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{6}\right) - 2\sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{3}\right)}{-\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{6}\right) - 2\cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{3}\right)} = \frac{\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{-\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)} = \frac{\sqrt{3}}{7}$$

The correct choice is (D).

18. $\dfrac{dy}{dt} = \dfrac{1}{2}(2t+5)^{-\frac{1}{2}} \cdot 2 = (2t+5)^{-\frac{1}{2}}$ and $\dfrac{dx}{dt} = 1 - 2t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \left.\frac{(2t+5)^{-\frac{1}{2}}}{1-2t}\right|_{t=2} = \frac{9^{-\frac{1}{2}}}{-3} = -\frac{1}{9}$$

The correct choice is (C).

35. Speed $= \sqrt{(x'(t))^2 + (y'(t))^2}$

$x(t) = 4\sin(\pi t)$ so $x'(t) = 4\pi\cos(\pi t)$

$y(t) = (3t-1)^2$ so $y'(t) = 2(3t-1)(3)$ or $18t - 6$

At $t = 2$, $x'(2) = 4\pi\cos(2\pi) = 4\pi$ and $y'(2) = 30$

Therefore, the speed at $t = 2$ is $\sqrt{(4\pi)^2 + (30)^2} = 32.526$

The correct choice is (C).

3. To find the vertical tangents, set $x'(t) = 0$.

$x'(t) = -4 \cos t$

$x'(t) = 0$ when $-4 \cos t = 0 \Rightarrow \cos t = 0$ or $t = \frac{\pi}{2}, \frac{3\pi}{2}$.

Vertical tangents occur at all points of the form

$\left(x\left(\frac{\pi}{2}\right), y\left(\frac{\pi}{2}\right)\right)$ and $\left(x\left(\frac{3\pi}{2}\right), y\left(\frac{3\pi}{2}\right)\right)$.

So the vertical tangents are at $(-1,4)$ and $(7,4)$.

Note: To find the horizontal tangents, set $y'(t) = 0$.

The correct choice is (C).

3. For the particle to be at rest both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ must both equal zero at the same time.

$\frac{dx}{dt} = t^3 - 3t^2 - 4t = t(t^2 - 3t - 4) = t(t+1)(t-4)$. Therefore $\frac{dx}{dt} = 0$ at $t = -1$, 0, and 4.

$\frac{dy}{dt} = t^3 - 16t = t(t^2 - 16) = t(t+4)(t-4)$. Therefore $\frac{dy}{dt} = 0$ at $t = -4$, 0, and 4.

Thus $\frac{dx}{dt}$ and $\frac{dy}{dt}$ both equal 0 at $t = 0$ and at $t = 4$, at which times the particle is at rest.

The correct choice is (A).

7. Statement I is false. Since $\int_0^3 f(x)\,dx$ equals a constant, $\frac{d}{dx}\int_0^3 f(x)\,dx = 0$ by the Fundamental Theorem of Calculus.

Statement II is false because $\int_3^x f'(x)\,dx = f(x) - f(3)$ (Fundamental Theorem of Calculus).

Statement III is a true representation of the Fundamental Theorem of Calculus.

The correct choice is (B).